Pre-Purchase Sampling and Quality of Experience Goods

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Abstract

We refine the classification of search goods and experience goods by the probability of the consumer sampling before purchase. If the consumer conducts a lot of searches before deciding to purchase, then the product is said to be more of a search good. If the consumer rarely or never searches before purchase, then the product is said to be more of an experience good. In a dynamic model where a monopolist firm sells to a sequence of short-lived consumers, reductions in search costs through a critical value induce consumers to search for information on product quality and encourage the firm to start producing high-quality products with positive probability. Further reductions in search costs decrease the frequency of consumer searches and increase the probability of the firm producing high-quality goods. The firm earns more profits if it absorbs all search costs instead of letting the consumers bear all costs.

1 Introduction

Classifying products into search and experience categories has been one of the major paradigms in economics and business research since the seminal work of Nelson (1970). Whereas much of the empirical literature on consumer behavior keeps testing methods to match actual products with the classification, (e.g. Huang et al. 2009, Gerard and Dion 2010, and Wan et al. 2012) its theoretical counterpart in the industrial organization has been discussing search and experience products in rather separate frameworks. The general practice is to assume that certain goods exogenously

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belong to the search or experience category, which means that consumers are already endowed with the information of product quality or that information acquisition is prohibitively costly so that consumers never search. This assumption implies that firms always exert the lowest possible effort on quality if the discount factor is below a certain threshold (e.g. Klein and Leffler 1981, Shapiro 1982, Dana and Fong 2011), which may not capture the whole truth of real-world market outcomes.

Noting quick advancements in information technologies, Klein (1998) was among the first to suggest that reduced sampling costs of information might have broken the wall between the search and experience categories. She proposed a new type of goods called "experience-cum-search" goods, of which qualities conventionally deemed as experiential become searchable through the Internet. She reasons that consumers would choose to search for product information prior to purchase whenever sampling costs are below the price of the product, and sample by experience otherwise. Her descriptive framework inspired a volume of empirical research on online marketing, for example, Bei et al. (2004), Huang et al. (2009), and Girard and Dion (2010). However, to the best of our knowledge, a formal model to examine how certain products may be reclassified and to explore the underlying mechanism of the reclassification has been absent. In this paper, we develop a theoretical model to analyze the effect of information sampling costs, among other factors, on the classification and quality outcome of a product. More specifically, we look at the role sampling costs play in the strategic interaction between consumers and firms as we relax the assumption that a product is exogenously assigned to the search or experience category.

We start by modeling a simple scenario with a firm producing and selling a product of either high or low quality to one consumer in a static setting. The firm makes decisions on both product quality and pricing, while the consumer decides whether or not to incur a positive sampling cost to inspect product quality before purchase. The possibility of pre-purchase information search gives rise to a perfect Bayesian mixed-strategy equilibrium in which the firm produces high-quality goods with a positive probability and the consumer searches for quality information with a probability less than one. Extending the model into a dynamic setting gives a more complete picture of how sampling costs and the discount factor jointly determine the market outcome.

Consistent with the literature, pure experience goods with high quality can be sustained in our model with high discount factors regardless of how costly it is to search. On the other hand, when the discount factor is low and sampling costs are high, only the low-quality equilibrium is sustainable. Our new result is, however, that for relatively low costs of information search, a mixed-strategy equilibrium emerges and brings the seller the highest expected profits when the pure strategy of producing high-quality goods is not sustainable. In the mixed-strategy equilibrium, the buyer must search for product information prior to purchase with a positive probability to ensure that the seller has the incentive to produce at high quality; the seller must produce at low quality with a positive probability to ensure the buyer has the incentive to search. In this equilibrium, the seller's quality is invariant to the discount factor, but a higher discount factor leads to a more efficient outcome because consumers conduct costly information search at a lower probability. As sampling cost decreases, the seller produces at high quality with a higher probability and the consumers search for quality information with a lower probability.

An important insight of our analysis is that the search-experience classification shall be interpreted as a continuous spectrum instead of discrete categories. If a consumer searches more frequently, then the goods shall be considered more of "search" goods. In our setting, pure search goods (in the sense that consumers always search before purchase) never exist for any market outcome since they do not constitute an equilibrium under the static setting. Even when such equilibrium exists in the dynamic model, it never maximizes profit for the firm and therefore is never preferred. On the other hand, pure experience status is still sustainable when the seller is sufficiently patient, for the benefit of maintaining a good reputation strictly dominates any incentive to cheat on the buyer in the current period.

One interesting implication of our model is that decreases in sampling costs have non-monotone effects on consumers' search behavior. Restricting attention to the case when the discount factor is not high enough to sustain an equilibrium in which consumers never search and the firm always produces high-quality goods, we find that a decrease in sampling costs initially leads to an upward jump in the probability of search as the mixed-strategy equilibrium replaces the low-quality equilibrium as the optimal equilibrium for the seller. Further decreases in the sampling costs lower the likelihood of consumer search continuously. This is because in the mixed-strategy equilibrium, a lower sampling cost allows the seller to charge a higher price for its product, which raises the value of reputation. In that case, even if the buyers search with a lower probability, the seller still has the incentive to produce at high quality more often. Such distinction between a switch of optimal equilibrium and changes in outcomes within the same class of equilibrium might help us resolve some seemingly conflicting empirical findings. For example, Ratchford et al. (2003) and Klein and Ford (2003) both study consumer information search within the context of the automobile industry. The former finds that younger consumers tend to search on the Internet, while the amount of search would have been larger if the Internet had not been present. In contrast, Klein and Ford (2003) find that the substitution of online media for traditional media increases the total amount of search. Our framework is able to accommodate both findings, with the interpretation that one corresponds to changing outcomes within the same type of equilibrium, and the other a shift in the type of optimal equilibrium.

Since this paper is related to a number of streams of literature, we review relevant work in the following section. The benchmark static model is presented in section 3 and adapted to dynamic settings in section 4. In section 5, we examine some extensions of the model. Section 6 concludes.

2 Literature Review

The classification of search and experience goods was first suggested by Nelson (1970). The literature of experience goods market and reputation mechanism branches from his work and the effect of imperfect product information on market outcomes has been extensively discussed. For example, Shapiro (1982) studies how reputation concern serves as an incentive for a monopolist to produce high quality products in a dynamic setting. Allen (1984) extends the study to a perfectly competitive environment and Shapiro (1983) discusses the case with continuous quality space. More recently, Horner (2002) and Kranton (2003) find opposite effects of competition on firms' incentive to maintain good reputations. Bar-Isaac (2005) and Dana and Fong (2011) find different non-monotone relationships between competition and quality. All of those articles assume that products are exogenously given as the experience type so that consumer search for information on quality is not discussed. Uncertainty on the production side is also studied, such as in Hörner (2002), Liebeskind and Rumelt (1989), and Mailath and Samuelson (2001) where firms may fail to produce high-quality goods even when they decide to exert the effort, or as in Gale and Rosenthal (1994) where a firm knows in advance when its business will terminate due to external shocks. Notably, Mailath and Samuelson (2001) prove that a mixed-strategy Markov perfect equilibrium exists but rule it out as implausible, whereas we show in this paper that a mixed-strategy equilibrium can be realized under certain conditions.

The vast majority of this literature agrees that producers have to be sufficiently patient, or that repeated sales should be sufficiently frequent, in order to realize high-quality goods provision in an equilibrium. Earlier studies, for example Klein and Leffler (1981) and Riordan (1986), as well as recent works such as Dana and Fong (2011) and Julien and Park (2014), mostly imply that high quality never realizes as an equilibrium outcome if the discount factor is lower than a cutoff value. However, we will show that it is possible for high quality to realize at any discount factor, as long as the sampling cost is small enough. Considering the technologies of this Internet age we live in, the case of moderate sampling cost is particularly relevant.

Another strand of literature studies consumer search as a mandatory procedure before purchase. Kohn and Shavell (1974) suggest a general model of search with the assumption that the distribution of product quality is exogenously determined. Diamond (1971) considers a dynamic model of price adjustment which contains some interaction between firms and consumers. Wilde (1980) proves the existence and uniqueness of the reservation price when a consumer samples products. More recent works by Liu (2011) and Liu and Skrzypacz (2014) study costly information acquisition with limited history in contexts similar to the experience goods market, but the experiential nature of goods is still taken as exogenous. This exogeneity of the goods' value to the consumers conitnues to be a common feature in this literature, as reflected in Choi et al. (2018), Chen et al. (2022), Chen (2022), and Moraga-Gonzalez and Sun (2023). To the best of our knowledge, this paper is the first that derives a mathematical framework to endogenize the interaction between a firm's reputation for its experience quality and pre-purchase sampling.

As mentioned earlier, Klein (1998) proposes that the classification of products may change over time when she reviews the literature in the context of the emergence of interactive media. She states that some experience products in the 70s may be reclassified as search goods if their "dominant or intrinsic attributes are now available to consumers prior to purchase" due to the rapid development of information technology. For example, microwave was classified as experience products by Nelson (1970) but reclassified as search goods in a recent survey by Girard and Dion (2010). However, empirical studies do not reflect that the reclassification is a one-way process. As Nelson (1970) classified cameras as search goods, Huang et al. (2009) have cameras, and Girard and Dion (2010) digital cameras, under the experience category. Such change is counter-intuitive under Klein's (1998) framework as it should not have become harder for consumers to search for information of cameras after decades have gone by. One explanation is, as Wan et al. (2012) point out, that heterogeneity among consumers may lead to different perceptions of the same product, and hence mixed consumer behaviors in the very same market. Our model presents another possibility, without resorting to heterogeneous consumers, that reduction in sampling costs leads to higher price premium and lower search frequency so that a product may be reclassified as experience goods.

Although Klein (1998) highlights the importance of the endogenous nature of whether a product shall be classified as search or experience, her analysis is only descriptive and mostly based on intuition. Qian et al (2015) is the only study we are aware of so far that puts searchable and experiential qualities in the same analytical model, in the context of how branded products respond to entries of counterfeit products. In their setting, the branded firms choose the levels of the searchable and experiential attributes, but consumers cannot decide whether to search for information on either attribute since they are exogenously designated to be searchable or experiential. In contrast, our setting allows for only one product attribute, but its nature is endogenously determined by consumers' behavior.

3 Benchmark Static Model

In this section, we illustrate the key idea of the mixed-strategy equilibrium in a static setting. A risk-neutral monopolist firm sells a single product to a risk-neutral consumer. The firm chooses to produce the product with low quality at cost c_l or with high quality at higher cost c_h. The consumer values low-quality product at v_l (greater than c_l) and high-quality at v_h (greater than v_l and c_h.) We assume that it is socially optimal to produce high-quality product, which means $v_h - c_h > v_l - c_l$. Moreover, we assume a reversible cost of production so that the firm suffers no loss if nothing is sold.

At the beginning of the game, the firm first decides simultaneously the price and quality of the good, with only the price being observable to the consumer. The consumer then decides whether or not to inspect product quality by incurring a positive sampling cost of z. We assume that the information obtained from the inspection is perfect. In addition, we assume that the consumer always purchases when she is indifferent between purchasing or not.

A trivial equilibrium exists with this setting, in which the consumer never inspects and the firm always sells low-quality goods at a price of v_l. We denote the probability for the consumer to inspect by σ, the product price by p, and the probability for the monopolist to produce high-quality product by q. Therefore, the trivial equilibrium is characterized by $(\sigma, p, q) = (0, v_l, 0)$.

We can also rule out some pure strategies directly. For example, if the buyer inspects with probability one, the seller would respond with producing high-quality goods with probability one. The buyer's best response would thus be not to inspect, and the seller's best response would in turn be to always exert low effect. This argument shows that the buyer would not always inspect and that the seller would not always exert high effort.

To construct a mixed-strategy equilibrium, we need to make sure that the consumer is indifferent between inspection and direct purchase and that the firm is indifferent between producing high- and low-quality goods. These two conditions require that

$$-z + q(v_h - p) = q(v_h - p) + (1-q)(v_l - p) \tag{1}$$

$$p - c_h = (1-\sigma)(p - c_l). \tag{2}$$

The left-hand side of equation (1) is the consumer's expected utility from inspection, and the right-hand side is that from direct purchase. The left-hand side of the second equation is the seller's profit when producing at high quality (so that the consumer would always buy) and the right-hand side is the expected profit from producing at low quality. When the seller exerts low effort, he will sell his product only if the buyer makes direct purchase with probability $1 - \sigma$.

Rearranging the two equations yields

$$p = v_l + \frac{z}{1-q} \tag{3}$$

$$\sigma = \frac{c_h - c_l}{p - c_l}. \tag{4}$$

Participation constraints of the consumer and the firm require that they are at least as good as in the trivial equilibrium, so that

$$-z + q(v_h - p) \geq 0 \tag{5}$$

$$p - c_h \geq v_l - c_l. \tag{6}$$

Equivalently,

$$p \leq v_h - \frac{z}{q}$$

$$p \geq c_h + v_l - c_l.$$

We restrict attention to the optimal equilibrium for the seller, or equivalently, an equilibrium with the highest $p - c_h$ possible. In the optimal equilibrium, the consumer's participation constraint (5) must bind, i.e. $p = v_h - z/q$, otherwise the seller could achieve higher profits. Suppose $p < v_h - z/q$ and $p = v_l + z/(1-q)$. Then by continuity, there exist $p' = p + \varepsilon_1$ and $q' = q + \varepsilon_2$ for $\varepsilon_1, \varepsilon_2 > 0$ such that $p' < v_h - z/q'$ and $p' = v_l + z/(1-q')$. In other words, the seller's profit can be improved from $p - c_h$ to $p' - c_h$, which constitutes a contradiction.

Consequently, we can solve the equilibrium with the incentive compatibility constraints (3) and (4) so that

$$p^* = \frac{1}{2}\left(v_h + v_l + \Delta\right),$$

$$q^* = \frac{1}{2}\frac{v_h - v_l + \Delta}{v_h - v_l}$$

$$\sigma^* = \frac{2(c_h - c_l)}{v_h + v_l - 2c_l + \Delta}.$$

where $\Delta = \sqrt{(v_h - v_l)(v_h - v_l - 4z)}$. Note that we retain only the set of solutions with greater prices as we are looking for the optimal equilibrium for the seller.

From this set of solution, we can see that the mixed-strategy equilibrium exists whenever $4z \leq v_h - v_l$. The firm would prefer this equilibrium if it generates higher expected profits than the trivial equilibrium when the constraint (6) is satisfied:

$$\frac{1}{2}\left(v_h + v_l + \sqrt{(v_h - v_l)(v_h - v_l - 4z)}\right) \geq c_h + v_l - c_l.$$

This inequality holds whenever

$$\frac{c_h - c_l}{v_h - v_l} \leq \frac{1}{2} \text{ and } z \leq \frac{v_h - v_l}{4},$$

or

$$z \leq \frac{c_h - c_l}{v_h - v_l}\left(1 - \frac{c_h - c_l}{v_h - v_l}\right)(v_h - v_l).$$

To summarize, the analysis of the static game gives us two equilibria. When sampling costs are high, the seller would only produce at low quality and set the price at v_l, while the buyer never inspects before purchase. The product would then be taken as experience goods since we observe that the buyer never searches. When sampling costs are low, however, the possibility of pre-purchase inspection gives rise to a mixed-strategy equilibrium that attracts the seller with higher expected profits. We would observe that the buyer searches for information of product quality before purchase, hence interpret the product as more of a kind of search goods.

4 Dynamic Model

To extend the model with dynamic settings, we consider a long-lived producer with discount factor $\delta \in (0, 1)$ and a sequence of consumers, each living for only one period. Consumers' beliefs about the firm's strategy is that whenever the firm is observed selling low-quality products, it will continue to do so in all remaining periods.

The timeline of a period is as follows. The history of product quality is publicly available at the beginning of a period. The firm first decides whether to produce with high or low quality and sets the price. The consumer then decides whether to inspect product quality or to buy without inspection. If the consumer chooses to inspect, product quality is revealed perfectly, and she may choose to leave without purchasing. Payoffs to the firm and the consumer are then realized, and the quality of the product becomes public information at the end of the period, regardless of the consumer's search behavior. If mixed strategies were played and the producer chose low quality, a public randomization device then reveals a public signal to coordinate the players' strategies in the next period.

We focus on perfect public equilibria (PPE) that lead to the highest return for the firm, which we refer to as *optimal equilibria.* Consumers' strategies are a function of the firm's current prices, beliefs about the firm's current product quality (which follows a function of the firm's past actions), and the realization of the public randomization device.

Before analyzing equilibria with inspection, we first briefly review the case without inspection.

4.1 Optimal Equilibria without Inspection

When it is impossible to inspect, there are two pure-strategy equilibria for the game, one in which only low-quality product is produced and the other in which only high-quality product is produced.

In the former case, consumers do not believe that high quality is ever provided in equilibrium so that purchases will not happen at any price higher than v_l. The firm, being able to set its highest price at v_l, maximizes its profit only when it produces low-quality product at a cost $c_l < c_h$. Since $v_l > c_l$, this equilibrium is sustainable under any circumstance. We call this the low-quality equilibrium (*LE*.)

In the other case, suppose the consumer's belief is that the firm produces high-quality goods at price v_h. If the price is set below v_h or product quality is ever revealed to be low in previous periods, the consumer immediately expects quality to be low forever. (This means that the low-quality equilibrium is used as the punishment path.) Since the current price is publicly observable, the firm deviates only in product quality if there is any incentive for it to deviate. Given the punishment path, the monopolist produces high-quality products in every period if and only if the benefit from deviating does not exceed the discounted future profit from selling high-quality products. Mathematically,

$$\frac{v_h - c_h}{1-\delta} \geq v_h - c_l + \delta \frac{v_l - c_l}{1-\delta},$$

or equivalently,

$$\delta \geq \frac{c_h - c_l}{v_h - v_l}.$$

We call this pure-strategy equilibrium the high-quality equilibrium (*HE*.) The condition to sustain this equilibrium has been characterized in literature with the following lemma.

Lemma 1 Monopoly Reputation

Suppose inspection is impossible. When a single firm sells a product with unobservable quality, if

$$\delta \geq \frac{c_h - c_l}{v_h - v_l} \equiv \delta_H,$$

then there exists a perfect Bayesian equilibrium in which the firm produces high-quality product and earns discounted profits of

$$\pi_H = \frac{v_h - c_h}{1 - \delta}.$$

Otherwise, the unique equilibrium outcome has low product quality and the price at v_l with discounted profits

$$\pi_L = \frac{v_l - c_l}{1 - \delta}.$$

4.2 Optimal Equilibria with Inspection

Now we introduce the possibility for consumers to inspect product quality before purchase in the dynamic setting. The two pure-strategy equilibria continue to exist under the same conditions. In these equilibria, consumers never choose to inspect because they have degenerate beliefs on quality that are consistent with the monopolist's strategy. Moreover, for $\delta \geq \delta_H$, the *HE* remains optimal for the seller because it is efficient and the seller captures all the surplus. With our notations, the *LE* and the *HE* are characterized by $(\sigma, p, q) = (0, v_l, 0)$ and $(\sigma, p, q) = (0, v_h, 1)$ respectively.

4.2.1 Mixed-Strategy Equilibrium

The focus of our analysis is to explore whether or not the possibility of pre-purchase inspection leads to an equilibrium that provides the seller with a higher profit than the *LE* does when the *HE* is not sustainable. In our case, it is possible to implement a mixed-strategy equilibrium (*ME*) because consumers get to choose whether or not to inspect before purchase, which makes their equilibrium decision to inspect and the firm's decision to produce high-quality products depend on each other.

We assume that consumers' equilibrium beliefs are that the firm produces high-quality goods with probability $q \in [0, 1]$ as long as the price is p given that the firm has always produced high-quality goods in the past. Low quality is expected when the price is set differently from p. When low quality is expected for sure, consumers will not buy at any price higher than v_l in the current period. Upon observation of low-quality goods, players are coordinated by the public randomization device to keep playing mixed strategies next period with probability ρ and to play the low-quality pure strategies next period with probability $1-\rho$. This randomization outcome forms the punishment path for the mixed-strategy equilibrium. Given such beliefs, the firm would not set the price different from p if it decides to produce with low quality, since consumers can immediately observe the price at no cost.

The set of the firm's payoff in equilibrium is an interval $[\underline{v}, \overline{v}]$ where $\underline{v}$ and $\overline{v}$ are the discounted values of the lowest and the highest possible profits, respectively. Since we assumed that product quality becomes public information at the end of each period independent from consumer search, there are four public outcomes denoted by $y \in Y \equiv \{0, 1, 2, 3\}$:

$$y = \begin{cases} 0 & \text{low quality, no inspection} \\ 1 & \text{low quality, inspection} \\ 2 & \text{high quality, no inspection} \\ 3 & \text{high quality, inspection} \end{cases}$$

The firm's value is represented by

$$\begin{aligned} v \equiv & (1-q)\left[(1-\sigma)\left(p-c_l+\delta v\left(0\right)\right)+\sigma\delta v\left(1\right)\right] \\ & +q\left[(1-\sigma)\left(p-c_h+\delta v\left(2\right)\right)+\sigma\left(p-c_h+\delta v\left(3\right)\right)\right] \end{aligned} \tag{7}$$

where $v(y)$ is the continuation value of the firm following the public outcome y.

As we focus on the optimal equilibrium for the firm, the maximum value of the firm is achieved by solving

$$\max_{p,q,\sigma,v(\cdot)} v$$

subject to

$$(p-c_h)+\delta[(1-\sigma)v(2)+\sigma v(3)] \geq (1-\sigma)(p-c_l)+\delta[(1-\sigma)v(0)+\sigma\delta v(1)] \tag{8}$$

$$p \in [v_l, v_h] \tag{9}$$

$$qv_h + (1-q)v_l - p \geq 0 \tag{10}$$

$$\sigma = \begin{cases} 1, & \text{for } q(v_h - p) - z > qv_h + (1-q)v_l - p \\ [0,1] & \text{for } q(v_h - p) - z = qv_h + (1-q)v_l - p \\ 0, & \text{for } q(v_h - p) - z < qv_h + (1-q)v_l - p \end{cases} \tag{11}$$

where the first inequality constraint imposes incentive compatibility for the firm to weakly prefer producing high-quality goods whenever $q > 0$, i.e. when the firm produces high quality goods with positive probability.

We refer the reader to the appendix for detailed steps of solving the problem, and characterize the result directly here.

Proposition 1 Optimal Equilibrium with Possibility of Inspection

1. *When* $\delta \geq \delta_H \equiv \dfrac{c_h - c_l}{v_h - v_l}$*, the Pure-strategy High-quality Equilibrium is optimal for the firm and* $(\sigma, p, q) = (0, v_h, 1)$.

2. *When* $\delta < \delta_H$*, the Mixed-strategy Equilibrium is optimal for* $z \in (0, \bar{z})$ *where*

$$\bar{z} \equiv \begin{cases} \dfrac{v_h - v_l}{4}, & \textit{for } \delta_H \leq \dfrac{1}{2}, \\ \delta_H(1-\delta_H)(v_h - v_l), & \textit{otherwise.} \end{cases}$$

The buyer inspects the product prior to purchase with probability

$$\sigma^* = \frac{2\left(c_h - c_l\right) - \delta(1-\rho)\left(v_h - v_l + \sqrt{\left(v_h - v_l\right)\left(v_h - v_l - 4z\right)}\right)}{\left(1 - \delta(1-\rho)\right)\left(v_h + v_l + \sqrt{\left(v_h - v_l\right)\left(v_h - v_l - 4z\right)} - 2c_l\right)} \in (0,1)\,,$$

which is increasing in z *and decreasing in* δ*. The buyer purchases if no inspection takes place or if product quality is high upon inspection. The seller produces high quality with probability*

$$q^* = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{z}{v_h - v_l}} \in \left(\frac{1}{2}, 1\right)$$

and charges

$$p^* = \frac{1}{2}\left(v_h + v_l + \sqrt{(v_h - v_l)(v_h - v_l - 4z)}\right)$$

for its product. The present discounted value of the seller's profits is $\dfrac{p^* - c_h}{1-\delta}$.

Proposition 1 shows us how sampling costs and the discount factor jointly determine the revealed property of the product, and how variations in their values affect the market outcomes of price and quality as summarized below.

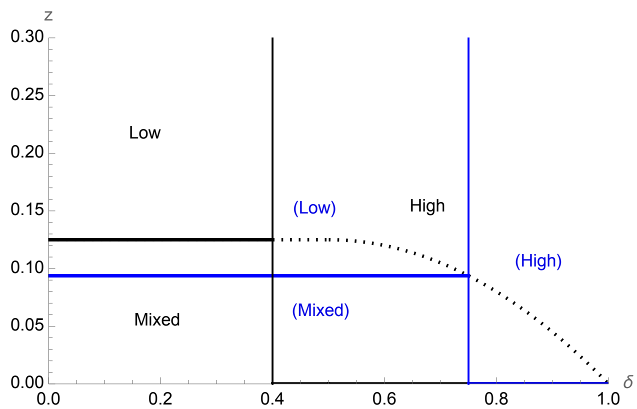

Figure 1: Illustration of Optimal Equilibrium Results

Corollary 1 ***Effect of sampling costs within mixed-strategy equilibrium*** *Product quality and price are both decreasing, while the frequency of consumer inspection increasing, in sampling costs when the mixed-strategy equilibrium is revealed as optimal.*

The positive relationship between sampling costs and the probability of searching in the mixed-strategy equilibrium may be counter-intuitive and deserves some explanations. When search becomes more costly, the consumer would have less incentive to search at the original price, which lowers the firm's incentive to produce high-quality goods as it becomes less likely to be spotted shirking on quality. However, this vicious cycle will result in a collapse into low-quality equilibrium. Therefore, it is in the firm's interest to set a lower price and invite the consumer to search more for the sake of its own expected profit.

From another perspective, the social surplus of producing one unit of high-quality goods is fixed, holding other factors constant. As the consumer inspects with positive probability, the social surplus is split between producer surplus and inspection costs, while consumer surplus remains zero as the participation constraint binds. When there is a reduction in inspection costs, consumers' participation constraints do not bind anymore so that the monopolist is able to capture this extra surplus by setting a higher price. This higher price makes the monopolist's future discounted profit more valuable and hence increases his willingness to produce high-quality goods. As a result, consumers do not have to inspect as often to keep the firm's incentive to produce with high quality. Therefore, when inspection costs become lower, the product becomes more of an experience good and less of a search good.

Corollary 2 *When $\delta < \delta_H$, the probability of high-quality provision jumps discretely from zero to one-half as sampling costs decrease through $\bar{z}$.*

This result is an illustration of how allowing for endogenous consumer search improves welfare. Without the possibility of inspection, consumers can only accept low-quality products when the discount factor is low. With inspection, however, improvement in welfare is significant at sufficiently low sampling costs.

Corollary 3 ***Effect of the discount factor within mixed-strategy equilibrium*** *Market outcomes of price and product quality are independent of the discount factor when the mixed-strategy equilibrium is revealed as optimal. Meanwhile, the frequency of consumer search is decreasing in the discount factor.*

Observing the expressions of equilibrium strategies in the *ME*, we find that the price and probability of producing a high-quality product are independent of the discount factor δ. This is because, in the mixed strategy equilibrium, the seller is able to charge a price that makes the buyer's expected payoff zero. For this reason, both the seller's probability of producing high quality and price are independent of the discount factor, and so is his average profit per period.

On the other hand, the consumer's probability to inspect is decreasing in δ in the *ME*. This means that *ceteris paribus*, a more patient seller would make the consumer inspect less frequently. When a seller decides on product quality, he is trading off between cost saving from shirking and losing higher future profit once caught. When he is more patient and values future profit more, the buyer does not have to inspect as often to maintain his incentive for quality improvement. In other words, when the seller becomes more patient, the product becomes more of an experience good and less of a search good.

5 Sharing the Sampling Costs

We have assumed that the consumer bears all the sampling costs until now, but it is often not the case in real life, especially for successful businesses. The rent Apple pays for its large stores and the return policies of Amazon are typical examples of how sellers absorb the sampling costs. We thus ask what the incentives are for firms to bear some sampling costs and what implications there may be.

We allow the seller to choose a fraction $\beta \in [0, 1]$ of the search cost that he will cover when the consumer chooses to inspect the product before purchase. In this case, the buyer's expected payoff from pre-purchase inspection becomes

$$q\left(v_h - p\right) - \left(1 - \beta\right) z \geq 0,$$

where the consumer incurs a cost of $\left(1 - \beta\right) z$ when she chooses to inspect. In a mixed-strategy equilibrium, the buyer's incentive compatibility constraint becomes

$$q\left(v_h - p\right) - \left(1 - \beta\right) z = q\left(v_h - p\right) + \left(1 - q\right)\left(v_l - p\right),$$

which implies the equilibrium relationship between price and quality:

$$p = v_l + \frac{\left(1 - \beta\right) z}{1 - q}.$$

Also, the participation constraint of the buyer is still binding:

$$q\left(v_h - p\right) - \left(1 - \beta\right) z = 0$$

so that equilibrium price satisfies

$$p = v_h - \frac{\left(1 - \beta\right) z}{q},$$

which means that

$$p = \frac{1}{2}\left(v_h + v_l + \sqrt{(v_h - v_l)\,(v_h - v_l - 4\,(1-\beta)\,z)}\right) \tag{12}$$

$$q = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{(1-\beta)\,z}{v_h - v_l}}. \tag{13}$$

Note that both the price and probability of producing high-quality product is increasing in the value of β. Therefore, the more the seller bears the sampling cost, the higher price he can charge and the greater social welfare there will be. Also, a necessary condition for this equilibrium to exist is

$$4\,(1-\beta)\,z \leq v_h - v_l,$$

which is a weaker condition compared with the corresponding condition in the mixed-strategy equilibrium without cost sharing. This means that, if the seller can freely alter his cost burden, he would be able to establish a mixed-strategy equilibrium at any level of sampling costs.

On the seller's side, he now has to consider his cost burden for his incentive:

$$\begin{aligned} v =& (1-q)[(1-\sigma)(p - c_l + \delta v(0)) + \sigma(-\beta z + \delta v(1))] \\ &+ q[(1-\sigma)(p - c_h + \delta v(2)) + \sigma(p - c_h - \beta z + \delta v(3))] \end{aligned}$$

which reduces to

$$v = \frac{p - c_h - \sigma\beta z}{1-\delta} \tag{14}$$

where

$$\sigma = \frac{c_h - c_l - \delta(p - v_l) + \delta\rho(p - c_l)}{(1 - \delta(1-\rho))(p - c_l) - \delta(1-\rho)\beta z} \tag{15}$$

Note that the buyer's probability to search is affected by the cost burden in two dimensions – the direct effect is that reduced burden on the buyer encourages more inspection, yet as the seller increases the price with a higher seller's burden on sampling cost, the probability of searching decreases with the price.

So far, we have derived the best strategies of the buyer and the seller as functions of β, the cost burden of the seller. Before characterizing the entire equilibrium, we first show that it is optimal for the seller to absorb all sampling costs.

Lemma 2 *When the producer can internalize the inspection costs, it will choose to absorb as much costs as possible. Mathematically,*

$$\frac{dv}{d\beta} > 0 \tag{16}$$

for any $(\beta, z) \in [0,1] \times \mathbb{R}^+$.

Proof. In this proof, we denote the optimal strategies and the seller's value as functions of β and z. The proof consists of two steps. First, we show that

$$\left.\frac{dv(\beta,z)}{d\beta}\right|_{z=0} = 0 \tag{17}$$

for all $\beta \in [0,1]$. Then we show that

$$\frac{d}{dz}\frac{dv(\beta,z)}{d\beta} > 0 \tag{18}$$

for all $z \geq 0$, so that

$$\frac{dv(\beta,z)}{d\beta} > 0 \tag{19}$$

for any $(\beta, z) \in [0,1] \times \mathbb{R}^+$.

The first step is straightforward. We can total differentiate the discounted value of the seller based on (14) so that

$$\begin{aligned} d(1-\delta)v(\beta,z) &= \frac{dp(\beta,z)}{d\beta}d\beta - \sigma(\beta,z)zd\beta - \frac{d\sigma(\beta,z)}{d\beta}\beta z d\beta \\ &= z\sqrt{\frac{v_h - v_l}{v_h - v_l - 4(1-\beta)z}} - \sigma(\beta,z)zd\beta - \frac{d\sigma(\beta,z)}{d\beta}\beta z d\beta \\ &= z\left(\sqrt{\frac{v_h - v_l}{v_h - v_l - 4(1-\beta)z}} - \sigma(\beta,z) - \frac{d\sigma(\beta,z)}{d\beta}\beta\right)d\beta. \end{aligned} \tag{20}$$

It is obvious that

$$\frac{dv(\beta,0)}{d\beta} = 0 \tag{21}$$

For positive values of z, equation (20) implies that $dv(\beta,z)/d\beta > 0$ if and only if

$$\sqrt{\frac{v_h - v_l}{v_h - v_l - 4(1-\beta)z}} - \sigma(\beta,z) - \frac{d\sigma(\beta,z)}{d\beta}\beta > 0. \tag{22}$$

Since the first term is greater than one for any positive z, it is sufficient to show

$$1-\sigma(\beta,z)-\frac{d\sigma(\beta,z)}{d\beta}\beta>0. \tag{23}$$

First, define

$$f(\beta,z)\equiv-\sigma(\beta,z)-\frac{d\sigma(\beta,z)}{d\beta}\beta. \tag{24}$$

Based on the solution of σ shown in (15), we obtain

$$f(\beta,z)=-\sigma(\beta,z)+\beta\left(\frac{Dp_{\beta}(\beta,z)+\sigma(\beta,z)[\tilde{D}p_{\beta}(\beta,z)-Dz]}{\tilde{D}[p(\beta,z)-c_l]-D\beta z}\right) \tag{25}$$

where $D\equiv\delta(1-\rho)$, $\tilde{D}\equiv 1-D$, and $p_{\beta}(\beta,z)\equiv dp(\beta,z)/d\beta$. This equation can be transformed into

$$f(\beta,z)=\frac{\beta p_{\beta}(\beta,z)[D+\sigma(\beta,z)\tilde{D}]-\sigma(\beta,z)\tilde{D}[p(\beta,z)-c_l]}{\tilde{D}[p(\beta,z)-c_l]-D\beta z}. \tag{26}$$

Note that $p_{\beta}(\beta,0)=0$ so that $f(\beta,0)=-\sigma(\beta,0)$ and $1-f(\beta,0)>0$ whenever the mixed-strategy equilibrium exists.[1] For positive values of z, we note that the numerator in (26) is increasing in z because $p_{\beta}(\beta,z)$ is increasing in z and $p(\beta,z)$ is decreasing in z. We also note that the denominator is decreasing in z. Therefore,

$$\frac{df(\beta,z)}{dz}>0 \tag{27}$$

for all $z>0$ and, consequently,

$$\frac{dv(\beta,z)}{d\beta}\propto\sqrt{\frac{v_h-v_l}{v_h-v_l-4\,(1-\beta)\,z}}-\sigma(\beta,z)-\frac{d\sigma(\beta,z)}{d\beta}\beta \tag{28}$$

$$\geq 1+f(\beta,z)>1+f(\beta,0)=1-\sigma(\beta,0)>0 \tag{29}$$

for all $(\beta,z)\in[0,1]\times\mathbb{R}^{+}$.

Therefore, the value of the seller would be strictly increasing in β whenever the mixed-strategy equilibrium exists. In other words, *the seller will have the incentive to share as much sampling*

[1]One can show from (15) that $\sigma(0)<1$ is equivalent to $p-c_h>\delta(v_l-c_l)$, which is implied by the seller's participation constraint.

costs as possible to maximize its own profits, i.e., the optimal share of sampling costs $\beta^* = 1$. ■

With the introduction of cost sharing, we need to characterize the conditions under which this new equilibrium is preferred to the low-quality one by the seller. The seller's participation constraint is now

$$p - c_h - \sigma\beta z \geq v_l - c_l,$$

or equivalently

$$z \leq \frac{p - c_h - v_l + c_l}{\beta\sigma}. \tag{30}$$

As we have shown in Lemma 2, it is optimal for the seller to absorb all sampling costs, i.e., $\beta^* = 1$. We can thus write the seller's participation constraint as

$$z \leq \bar{z}_{CS} \equiv \frac{p^*_{CS} - c_h - v_l + c_l}{\sigma^*_{CS}}, \tag{31}$$

where p^*_{CS} and σ^*_{CS} denote the equilibrium price level and sampling probability. Given the optimal share of sampling cost $\beta^* = 1$, we can substitute p^*_{CS} and σ^*_{CS} into Equation (31) and obtain

$$\bar{z}_{CS} = \frac{\tilde{D}(v_h - c_l)(v_h - c_h - v_l + c_l)}{\delta\rho(v_l - c_l) + \tilde{D}(c_h - c_l)}$$

Proposition 2 Optimal Equilibrium with Shared Costs of Pre-Purchase Inspection

1. *When $\delta \geq \delta_H$, the Pure-strategy High-quality Equilibrium is optimal for the firm and $(\sigma, p, q) = (0, v_h, 1)$. Since the buyer never inspects, the firm does not need to specify the cost-sharing policy β. The present discounted value of the firm is $\frac{v_h - c_h}{1-\delta}$.*

2. *When $\delta < \delta_H$, the Mixed-Strategy Equilibrium is optimal for $z \in (0, \bar{z}_{CS})$ where*

$$\bar{z}_{CS} = \frac{(1 - \delta(1-\rho))(v_h - c_l)(v_h - c_h - v_l + c_l)}{\delta\rho(v_l - c_l) + (1 - \delta(1-\rho))(c_h - c_l)}.$$

The buyer inspects the product before the purchase with probability

$$\sigma^*_{CS} = \frac{c_h - c_l - \delta(v_h - v_l) + \delta\rho(v_h - c_l)}{(1 - \delta(1-\rho))(v_h - c_l) - \delta(1-\rho)z},$$

which is increasing in z. The buyer purchases if there is no inspection or if product quality is *high upon the inspection. The seller produces high-quality products for sure, i.e.,*

$$q^*_{CS} = 1,$$

charges the price

$$p^*_{CS} = v_h,$$

and absorbs all inspection costs incurred by the buyer, i.e.,

$$\beta^* = 1.$$

The present discounted value of the seller's profits is

$$v_{CS} = \frac{v_h - c_h - \sigma^*_{CS} z}{1 - \delta}.$$

6 Concluding Remarks

With a model of the repeated game between a monopolist firm and a consumer, we show that the possibility for the consumer to engage in a costly search for product quality enables us to characterize a mixed-strategy equilibrium in which goods of high quality are supplied with a high probability bounded below by one-half. When the consumer is willing to sample the product for quality with positive probability, the firm's effort in high-quality production has a chance to be discovered and higher profit can be realized. For sampling costs sufficiently low, the firm prefers this new mixed-strategy equilibrium to the conventional low-quality one found in the literature. Meanwhile, efficiency is improved with this new class of equilibrium as high-quality products are supplied with higher frequency.

Our results provide some new insights with regard to the relationship between the consumer behavior of search and the level of sampling costs. In the mixed-strategy equilibrium, product price, and quality are decreasing in sampling costs, which is an intuitive result; however, the frequency of consumer inspection increases with sampling costs. Although this is a result from the characteristics of the mixed-strategy equilibrium, there might be some empirical interests to verify whether this finding aligns with consumer behavior in the real world.

The relationship between consumer behavior and the classification of products shall also be re-examined when we look at the characterization of the mixed-strategy equilibrium. It is not the case, as described in Klein (1998), that "consumer will always sample goods" when sampling costs are lower than the market price of the product. In fact, consumers will never search with certainty, as we show that the optimal frequency in the mixed-strategy equilibrium is bounded away from one. Although an equilibrium exists in which the consumer always searches and the firm always produces high-quality goods, such a strategy sacrifices too much profit for the firm to prefer it over the mixed-strategy equilibrium. The frequency of consumer search decreases as sampling costs go down, and can be very low although bounded away from zero. This means that two different kinds of search goods can have very different market outcomes in terms of the frequency of consumer inspection before purchase. Put in a different way, the quality of search goods can rely much on reputation that resembles attributes of experience goods when the consumer searches with extremely low frequency. This implication shall alert future empirical research to avoid categorizing the products purely based on the frequency of consumer search.

Another finding against conventional wisdom is about the cutoff level of sampling costs for distinguishing search goods and experience goods. We show that the maximum level of sampling costs acceptable for consumers to conduct pre-purchase inspection is much lower than the price of the product, unlike what has been stated or implied in the literature that a consumer searches as long as sampling costs are lower than price (Klein 1998 and Nelson 1970.)

As we extend the analysis, we find that the firm has the incentive to absorb the sampling costs as much as possible so that it can post higher prices and hence earn higher profits. This is because the firm has the incentive to lower the consumer's sampling frequency by more frequently providing high-quality goods. When the firm absorbs all sampling costs, the mixed-strategy equilibrium can be sustained, for moderate sampling costs, with the firm *always* producing high-quality products.

This model investigates only a simple model of endogenous consumer search with a monopolistic market structure and a single consumer. It can be enriched by examining the market outcomes under different market structures, with heterogeneous consumers, or with continuous product quality space. Previous literature on experience goods has shown that these factors have important implications to market outcomes, and we expect that richer results emerge with the suggested modifications to the model.

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Appendices

A Solving for the Dynamic Optimal Equilibrium

Recall that the four public outcomes are denoted

$$y = \begin{cases} 0 & \text{low quality, no inspection} \\ 1 & \text{low quality, inspection} \\ 2 & \text{high quality, no inspection} \\ 3 & \text{high quality, inspection} \end{cases}$$

and the firm's problem is

$$\begin{aligned} \max_{p,q,\sigma,v(\cdot)} v \equiv & (1-q)\left[(1-\sigma)(p-c_l+\delta v(0))+\sigma\delta v(1)\right] \\ & + q\left[(1-\sigma)(p-c_h+\delta v(2))+\sigma(p-c_h+\delta v(3))\right] \end{aligned} \tag{A.1}$$

subject to

$$(p-c_h)+\delta[(1-\sigma)v(2)+\sigma v(3)] \geq (1-\sigma)(p-c_l)+\delta[(1-\sigma)v(0)+\sigma v(1)] \tag{A.2}$$

$$p \in [v_l, v_h] \tag{A.3}$$

$$qv_h+(1-q)v_l-p \geq 0 \tag{A.4}$$

$$\sigma = \begin{cases} 1, & \text{for } q(v_h-p)-z > qv_h+(1-q)v_l-p \\ [0,1], & \text{for } q(v_h-p)-z = qv_h+(1-q)v_l-p \\ 0, & \text{for } q(v_h-p)-z < qv_h+(1-q)v_l-p \end{cases} \tag{A.5}$$

where inequality (A.2) is the incentive compatibility constraint for the firm to weakly prefer producing high-quality goods whenever $q \geq 0$.

Since the game to be played upon observation of low-quality goods depends on the realization of the public randomization device, $v(0) = v(1) = \underline{v}$. On the other hand, note that increasing the values of $v(2)$, and $v(3)$ improves the expected payoff and also relaxes the constraint (A.2), so it is optimal to have $v(2) = v(3) = \overline{v}$. Substituting these relationships into (A.1), the expected payoff becomes

$$v = (1-q)[(1-\sigma)(p-c_l) + \delta\underline{v}] + q[(p-c_h) + \delta\overline{v}] \tag{A.6}$$

with the constraint (A.2) becoming

$$\delta(\overline{v} - \underline{v}) \geq (1-\sigma)(c_h - c_l) - \sigma(p - c_h). \tag{A.7}$$

By the usual argument, consumer's participation constraint (A.4) must bind, otherwise higher profit for the firm can be realized by increasing the price by an infinitesimal amount. Now that (A.4) binds, the constraint (A.5) is simplified to

$$\sigma = \begin{cases} 1, & \text{for } q(1-q)(v_h - v_l) - z > 0 \\ [0,1], & \text{for } q(1-q)(v_h - v_l) - z = 0 \\ 0, & \text{for } q(1-q)(v_h - v_l) - z < 0 \end{cases} \tag{A.8}$$

Suppose $q(1-q)(v_h - v_l) - z > 0$ so that $\sigma = 1$. It would then be optimal to set $q = 1$. However, this leads to a contradiction because $q(1-q)(v_h - v_l) - z|_{q=1} = -z < 0$. Alternatively, suppose that $q(1-q)(v_h - v_l) - z < 0$ so that $\sigma = 0$. The firm's value in (A.6) becomes

$$v' = qv_h + (1-q)v_l - [qc_h + (1-q)c_l] + (1-q)\delta\underline{v} + q\delta\overline{v}.$$

The optimal value of q depends on whether the value v' is increasing in q. Taking the derivative with respect to q, we have

$$\frac{\partial v'}{\partial q} = \delta(\overline{v} - \underline{v}) + v_h - v_l - c_h + c_l.$$

Since $\overline{v} \geq \underline{v}$ and it is assumed that $v_h - v_l - c_h + c_l > 0$, the partial derivative is always positive. It is thus optimal to have $q = 1$, $p = v_h$, and $v = v_h - c_h + \delta\overline{v}$. Since $\overline{v}$ is the only continuation value in v, the optimal value must be $v = \overline{v} = (v_h - c_h)/(1-\delta)$. In this case, the constraint (A.7) requires that $\delta \geq (c_h - c_l)/(v_h - v_l) = \delta_H$. This result aligns with Lemma 1 which characterizes a high-quality equilibrium. If the constraint (A.7) is not satisfied, then the firm would never prefer producing high-quality goods. Now suppose that $q(1-q)(v_h - v_l) - z = 0$ so that $\sigma \in [0,1]$. By this condition,

$$q = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{z}{v_h - v_l}}$$

and thus

$$p = \frac{1}{2}(v_h + v_l) \pm \sqrt{\frac{1}{4} - \frac{z}{v_h - v_l}}(v_h - v_l)$$

so it is necessary that $4z < v_h - v_l$. Taking partial derivatives of equation (A.6) with respect to q and p, we have

$$\frac{\partial v}{\partial q} = \sigma p + (1-\sigma)c_l + \delta(\overline{v} - \underline{v}) > 0$$

and

$$\frac{\partial v}{\partial p} = (1-q)(1-\sigma) + q > 0.$$

Therefore, it is optimal to have larger values of p and q, so that

$$p = \frac{1}{2}(v_h + v_l) + \sqrt{\frac{1}{4} - \frac{z}{v_h - v_l}}(v_h - v_l) \text{ and } q = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{z}{v_h - v_l}}.$$

Observing (A.6), we can see that the value is decreasing in σ. The right-hand side of (A.7) can be written as $c_h - c_l - \sigma(p - c_l)$, which is also decreasing in σ. Ideally, we would want to set $\sigma = 0$, which gives

$$v = q(v_h - c_h + \delta\overline{v}) + (1-q)\underline{v} < \frac{v_h - c_h}{1-\delta}.$$

As we look for the optimal equilibrium for the firm, we can take v as the maximum value $\overline{v}$ and obtain

$$\overline{v} = \frac{1}{1-\delta q}(q(p - c_h) + (1-q)\underline{v})$$

which in turn gives

$$\overline{v} - \underline{v} = \frac{q}{1-\delta q}(v_h - c_h - v_l + c_l)$$

so that the constraint (A.7) is equivalent to

$$\delta \geq \frac{c_h - c_l}{q(v_h - v_l)} > \frac{c_h - c_l}{v_h - v_l}.$$

Since the value of the firm using mixed strategies is lower and the set of feasible discount factors is smaller than those in the high-quality equilibrium, this mixed-strategy equilibrium is not optimal for the producer with a zero equilibrium probability to search ($\sigma = 0$). However, when $\sigma > 0$, it is possible to extend this equilibrium to $\delta < \delta_H$ and make it more profitable than the low-quality equilibrium. When $\sigma > 0$, the constraint (A.7) becomes

$$\delta \geq \frac{c_h - c_l - \sigma(p - c_l)}{\overline{v} - \underline{v}}.$$

Since the value of the firm is decreasing in σ, it is optimal to have

$$\delta = \frac{c_h - c_l - \sigma(p - c_l)}{\overline{v} - \underline{v}} < \frac{c_h - c_l}{v_h - v_l}$$

so that

$$\sigma = \frac{c_h - c_l - \delta(\overline{v} - \underline{v})}{p - c_l}.$$

Substituting σ into (A.6), we have

$$v = p - c_h + \delta\overline{v}$$

and the upper bound $\overline{v}$ can be obtained as

$$\overline{v} = \frac{p - c_h}{1 - \delta} = \frac{1}{1 - \delta}\left(\frac{1}{2}(v_h + v_l) + \sqrt{\frac{1}{4} - \frac{z}{v_h - v_l}}(v_h - v_l) - c_h\right).$$

Plugging $\overline{v}$ back for σ gives

$$\sigma = \frac{c_h - c_l - \delta(p - v_l)}{(1 - \delta)(p - c_l)}.$$

It is required that $\delta < (c_h - c_l)/(p - v_l)$ to have $\sigma > 0$. Since $p < v_h$, this constraint is always satisfied whenever $\delta < \delta_H$. For σ to be less than 1, it must be that $p > c_h + \delta(v_l - c_l)$, which always holds since $\overline{v} > \underline{v}$. To verify that $\overline{v} > \underline{v}$, we note that

$$\underline{v} = \rho\overline{v} + (1 - \rho)(v_l - c_l + \delta\underline{v}) = \frac{\rho\overline{v} + (1 - \rho)(v_l - c_l)}{1 - \delta(1 - \rho)} \tag{A.9}$$

so that $\overline{v} > \underline{v}$ implies

$$\overline{v} > \frac{v_l - c_l}{1 - \delta}, \tag{A.10}$$

which is equivalent to $p - c_h > v_l - c_l$. It turns out that the inequality is satisfied when either of the following conditions holds:

$$\delta_H \equiv \frac{c_h - c_l}{v_h - v_l} < \frac{1}{2} \tag{A.11}$$

$$z < \delta_H(1 - \delta_H)(v_h - v_l). \tag{A.12}$$